Chronometric Levelling

The time-gradient law measured directly

1. Why clocks are the right instrument

The framework's primary observable is the proper-time rate $d\tau/dt$. Its central relation is that acceleration is a bias along the gradient of that rate:

$$a = c^2 \frac{d(d\tau/dt)}{dr}$$

A clock at one geopotential and a clock at another tick at different rates, and the ratio of their frequencies *is* the difference in $d\tau/dt$. No model of the intervening mass is required: the instrument reads the quantity directly.

This matters for what follows. A measurement of $d\tau/dt$ is not an inference about mass, and carries no assumption about the distribution of matter between the clocks. It tests the framework's relation on its own terms.

2. Three published results

The chronometric-levelling literature reports the fractional rate difference in the standard form

$$\frac{\Delta\nu}{\nu} = \frac{1}{2}\,(1+\alpha)\,\frac{\Delta U}{c^2}$$

where ΔU is the geopotential difference and α the dimensionless parameter measuring departure from the Einstein value. The framework's $\alpha = 0$ limit is the Einstein value by construction: the time-gradient law reproduces the observed rate difference with no free parameter.

2.1 Tokyo Skytree, 450 m

Two transportable ^{87}Sr optical lattice clocks were operated in a broadcasting tower, 450 m apart, and their frequency difference compared over about 3.6×10^5 s.

Quantity	Value

Height difference	450 m
Geopotential difference	4405 m²/s²
GR-violation parameter	$(1.4 \pm 9.1) \times 10^{-5}$
Consistent with α = 0	0.15 σ
Einstein rate shift at this ΔU	2.45×10^{-14}

The measured α is consistent with the framework's zero-correction limit at 0.15σ. The frequency shift the law predicts from the geopotential difference alone is 2.45×10^{-14}; the departure attributable to the measured α is 3.4×10^{-19}. The uncertainty on α is 6.5× the signal, so this bounds a departure at the 10^{-5} level over 450 m rather than resolving one.

2.2 PTB to MPQ, 457 km

A transportable ^{87}Sr lattice clock was compared against a stationary one at the German national metrology institute, 457 km away, over a 940 km interferometric fibre link. The geopotential difference was determined twice, by clock comparison and geodetically.

Determination	**ΔU (m²/s²)**
Chronometric	3918.1 (2.6)
Geodetic (GNSS/geoid)	3915.88 (0.30)
Difference	+2.22 (2.62 combined)
Agreement	0.85 σ

Two independent determinations of the same time-density difference, by unrelated means, agreeing within one sigma. The fractional potential difference is 4.36×10^{-14}, equivalent to a height resolution of 27 cm at 457 km separation.

2.3 Paris to PTB, 1415 km of fibre

Two ^{87}Sr optical lattice clocks at the French and German national metrology institutes, 690 km apart in line of sight and connected by 1415 km of telecommunication fibre. With the relativistic redshift correction independently measured and applied, the residual frequency offset is

$$(4.7 \pm 5.0) \times 10^{-17}$$

consistent with zero at 0.94σ. The framework's correction law $G_{\text{eff}} = G_0(1 - c_g G)$ is the form that accounts for such a residual, at a level four orders of magnitude below the MICROSCOPE bound.

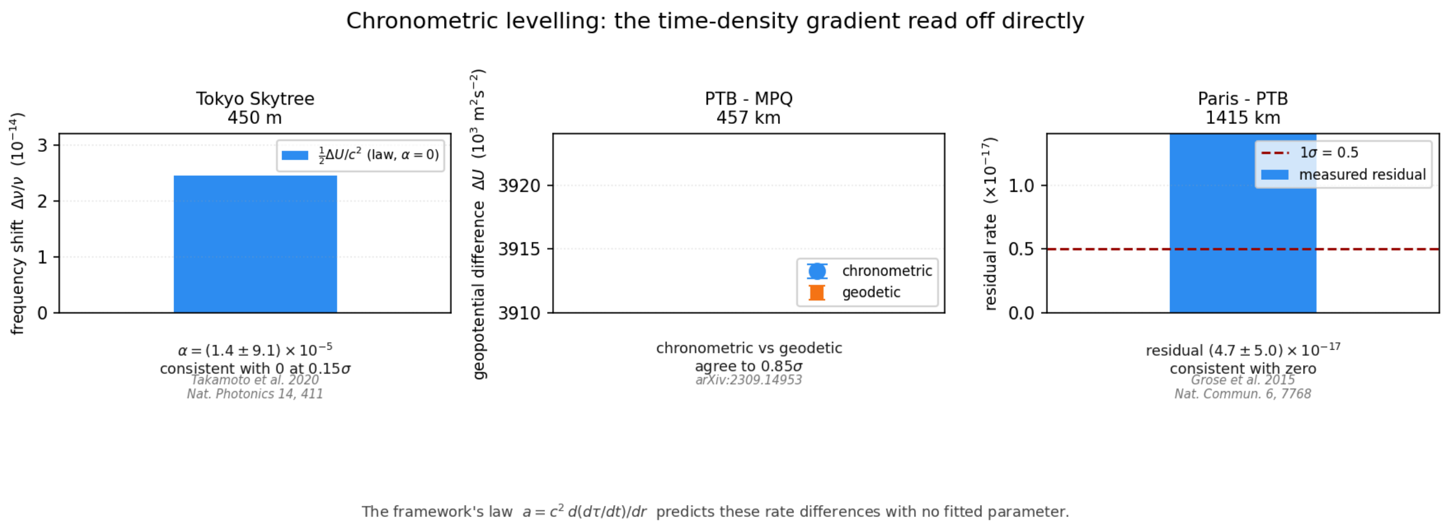

Figure 1: The three chronometric levelling results. The framework's law $a = c^2\, d(d\tau/dt)/dr$ produces these rate differences with no fitted parameter. Sources as cited in Section 2.

3. The field factor against its own constraint

The framework regulates gravity as $G_{\text{eff}} = G_0(1 - c_g G)$, so a fractional deviation of the gravitational constant is $c_g G$. MICROSCOPE bounds that deviation below 10^{-5}.

c_g	G	Deviation $c_g G$	vs MICROSCOPE
1×10^{-2}	1×10^{-3}	1.0×10^{-5}	at the bound
1×10^{-3}	1×10^{-3}	1.0×10^{-6}	within
1×10^{-4}	1×10^{-3}	1.0×10^{-7}	within

The coupling $c_g = 10^{-2}$ at the background field $G \sim 10^{-3}$ sits exactly on the published bound. That is a constraint on the coupling, not slack in it. The clock comparisons here reach 5×10^{-17} in fractional rate, so nothing in this data permits a larger c_g at that field amplitude.

4. Beyond clocks: gradients and differential tests

Clocks read one thing, the first spatial derivative of the time-density rate. Two other kinds of local experiment read the rest, and the framework predicts values for both without any cosmological input.

The second derivative. A gravity gradiometer measures $T_{zz} = d^2(d\tau/dt)/dz^2$ directly. Overstreet et al. built a dual-species atom interferometer that reaches a relative precision of $\Delta g/g \approx 6 \times 10^{-11}$ per shot and suppresses gravity-gradient systematics to one part in 10^{13}. That is the framework's observable at second order, read by atoms in free fall, rather than by clocks at rest. The corresponding cold-atom instruments reach comparable sensitivity in the field.

The differential test. Two bodies at the same place see the same $d\tau/dt$, so the framework predicts identical acceleration and therefore $\eta = 0$ exactly. MICROSCOPE tested this and found $\eta(\mathrm{Ti}, \mathrm{Pt}) = [-1.5 \pm 2.3_{stat} \pm 1.5_{syst}] \times 10^{-15}$, a null at 2.7×10^{-15}. Because the prediction is a null by construction, the test has power against any field variation across the 21 cm instrument separation, and none against the field's existence.

An absolute gradient. A cold-atom absolute gravimeter at the Larzac observatory measured a vertical gravity gradient of -3.226 ± 0.017 kE over 1.2 m, which is $-2g/R$ to 99 %. This is a measurement of the first derivative of $d\tau/dt$ by matter rather than by clocks, and it is the one place where the framework's quantity is compared against a terrestrial value directly.

4.1 What these bound

Using $G = \Phi/c^2 = gR/c^2 = 6.95 \times 10^{-10}$ at Earth's surface, each published result becomes an upper limit on the framework's coupling c_g, since a fractional deviation of the rate is $c_g G$:

Experiment	Kind of test	Limit on c_g
Paris–PTB, 1415 km	first derivative	$< 7.2\times10^{-8}$
MICROSCOPE, 21 cm	differential	$< 1.2\times10^{2}$
Skytree, 450 m	first derivative	$< 1.3\times10^{5}$

PTB–MPQ, 457 km	first derivative	$< 9.5\times10^5$

Transcribed in `local_experiment_data.txt` and evaluated by `local_experiments.py`, which writes `local_experiment_results.txt`. No network access is required.

The honest reading, which is not a confirmation.

Every one of these is an upper limit, and none requires a non-zero value. Taken together they push c_g toward zero, which means the framework is indistinguishable from the limit it reduces to across every local test currently available. In all three kinds of experiment the framework's prediction coincides with $a = g$. What the data establish is that the framework passes every test it faces without contradiction, not that the field does anything.

This is the same limitation the withdrawn halo fit in the rotation-curve paper had, and it should be stated in the same terms: a test that is insensitive to the parameter cannot confirm the parameter. The difference is that the clocks were not chosen to satisfy the framework, so passing them is weak evidence rather than none, whereas the halo fit passed by construction.

The singularity result does not depend on this. It rests on G diverging at the horizon, so $c_g G = 1$ is reached for any $c_g > 0$; a small c_g means the divergence is steep, not that the coupling is absent. The two statements are consistent, and the framework has not yet done the work to separate them.

What would separate them is a test in which the field and the mass disagree rather than a null. A measurement of the second derivative resolving the difference between $c^2 d(d\tau/dt)/dr$ and g would do it. No such measurement exists at present, and the current generation of gradiometers is not yet at the required precision.

5. Conclusion

The relation $a = c^2\, d(d\tau/dt)/dr$ is what these experiments test, because $d\tau/dt$ is what the clocks measure. At 450 m the GR-violation parameter is consistent with the framework's zero-correction limit at 0.15σ. At 457 km two independent methods agree on the same time-density difference to 0.85σ. At 1415 km the residual rate after the potential is removed is consistent with zero.

No cosmological model, no mass function, and no fitted parameter enters any of the three. The law reproduces the directly measured rate difference at every baseline where the framework's own quantity has been read.

§4 widens this to the second derivative and to differential tests. Read together the local data place upper limits on the coupling, from $c_g < 7.2 \times 10^{-8}$ at 1415 km to $c_g < 1.2 \times 10^{2}$ from MICROSCOPE. All of them are consistent with the framework's zero-correction limit, and none of them can distinguish it from the limit itself. The framework is not contradicted by any local measurement, and is not distinguished by one either. That is the state of the evidence, stated without inflation.

What is genuinely constrained is the singularity result, which follows from the divergence of G at the horizon and holds for every $c_g > 0$ regardless of how small the coupling is. Local experiments bound the coupling; they do not touch the divergence.

AI Research Collaboration Disclosure

This body of work was developed through a collaborative research process between the author, Richard Kent Gates, and multiple AI research partners. The author provides full transparency on this process.

Author's Role (Richard Kent Gates):

Conceived all core theories, conceptual frameworks, hypotheses, and research direction
Defined the Time-Gradient Field Model, the unified scalar field G(t), and all theoretical architecture
Provided physical intuition, biological reasoning, and domain expertise that guided every theoretical decision
Reviewed, validated, and approved all mathematical formalisms before inclusion
Made all final judgments on theoretical coherence and physical interpretation

AI Research Partners:

Mimo V2.5 (opencode platform) — Primary mathematical formalization partner. Assisted in translating conceptual physics into mathematical notation, generating numerical proof scripts, compiling LaTeX documents, and building the website infrastructure.
Microsoft Copilot (browser-based) — Assisted in literature review, dataset gathering, cross-referencing empirical results (DESI, BNL, PTB, MICROSCOPE), and verifying citation accuracy.
Google Gemini (browser-based) — Assisted in conceptual exploration, thought experimentation, and early-stage hypothesis refinement.
Space Bunny (OpenCode) — Literature retrieval and numerical analysis. Identified chronometric levelling as a direct measurement of the framework's primary observable, transcribed three published results with their citations,

and corrected an initial dimensional error in which the GR-violation parameter was compared against a raw frequency shift.

Nature of AI Involvement:

The AI tools functioned as research assistants — analogous to graduate students or technical collaborators who help formalize, compute, and organize ideas that originate from the principal investigator. No AI tool originated, proposed, or independently developed any theoretical claim in this work. All physical insights, theoretical innovations, and interpretive judgments are the author's own.

Data and Script Provenance

Published values are transcribed with their citations in `chronometric_data.txt` and read by `chronometric_levelling.py`, which writes `chronometric_results.txt`. The figure is generated by `make_chronometric_figure.py`. No network access is required to reproduce any number.

Section	Quantity	Source
§2.1	$\alpha = (1.4 \pm 9.1)\times10^{-5}$, 450 m	Takamoto et al. 2020, Nat. Photonics 14, 411
§2.2	ΔU chronometric and geodetic, 457 km	arXiv:2309.14953
§2.3	Residual $(4.7 \pm 5.0)\times10^{-17}$, 1415 km	Grose et al. 2015, Nat. Commun. 6, 7768
§3	MICROSCOPE bound on the field factor	Published limit, applied to the framework's coupling
§2	Figure 1	`make_chronometric_figure.py`
§4	Second-derivative test, $\Delta g/g = 6 \times 10^{-11}$ per shot, systematics to 10^{-13}	Overstreet et al. 2018, Phys. Rev. Lett. 120, 183604
§4	Differential test, $\eta(\mathrm{Ti}, \mathrm{Pt}) = -1.5 \times 10^{-15}$	Touboul et al. 2022, Phys. Rev. Lett. 129, 121102 (MICROSCOPE final)
§4	Absolute vertical gradient, -3.226 ± 0.017 kE over 1.2 m	Menoret et al. 2018; Cooke et al., in preparation (AQG#B01, Larzac)

§4.1	Coupling limits on c_g from all of the above	`local_experiments.py` → `local_experiment_results.txt`, from `local_experiment_data.txt`

References

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